HRI-P-06-11-005
CU-PHYSICS-18-2006

Neutrino parameters from matter effects in P_{ee} at long baselines

Sanjib Kumar Agarwalla[1,2], Sandhya Choubey[1], Srubabati Goswami[1], and Amitava Raychaudhuri[1,2]

[1] *Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211 019, India*
[2] *Department of Physics, University of Calcutta, 92 Acharya Prafulla Chandra Road, Kolkata 700 009, India*

(Dated: November 29, 2006)

We show that the earth matter effects in the $\nu_{\rm e} \to \nu_{\rm e}$ survival probability can be used to cleanly determine the third leptonic mixing angle θ_{13} and the sign of the atmospheric neutrino mass squared difference, Δm^2_{31}, using a β-beam as a ν_e source.

PACS numbers: 14.60.Pq, 14.60.Lm, 13.15.+g

One of the major aims of neutrino physics is to reconstruct the neutrino mass matrix. Three missing links in this program are the mixing angle θ_{13}, the sign of $\Delta m^2_{31} \equiv m^2_3 - m^2_1$ ($sgn(\Delta m^2_{31})$) and the CP phase δ_{CP}. The $\nu_e \to \nu_\mu$ transition probability $P_{e\mu}$, being dependent on all these three parameters, has been identified as the "golden channel" [1] for measuring δ_{CP}, $sgn(\Delta m^2_{31})$ and θ_{13} in long baseline accelerator based experiments. However, this strength of the golden channel also brings in the well-known problem of parameter "degeneracies", where one gets multiple fake solutions in addition to the true one [2]. Various ways to combat this vexing issue have been suggested in the literature, including combining the golden channel with the "silver" ($P_{e\tau}$) [3] and "platinum" ($P_{\mu e}$) channels. While each of them would have fake solutions, their combination helps in beating the degeneracies since each channel depends differently on δ_{CP}, $sgn(\Delta m^2_{31})$ and θ_{13}. In this letter, we propose using the $\nu_e \to \nu_e$ survival channel P_{ee}, which is *independent* of δ_{CP} and the mixing angle θ_{23}. It is therefore *completely* absolved of degeneracies and hence provides a clean laboratory for the measurement of $sgn(\Delta m^2_{31})$ and θ_{13}. This gives it an edge over the conversion channels, which are infested with degenerate solutions.

The P_{ee} survival channel has been extensively considered for measuring θ_{13} with $\bar{\nu}_e$ produced in nuclear reactors [4] and with detectors placed at a distance $L \simeq 1$ km from the source. Reducing systematic uncertainties at the sub-percent level is a prerequisite for this program and enormous R&D is underway for this extremely challenging job. In the context of accelerator based experiments, the survival channel P_{ee} has been discussed with sub-GeV energy neutrinos from a β-beam source at CERN and a megaton water detector in Frejus at a baseline of 130 km [5]. However, since systematic uncertainties for such an experimental set-up is expected to be much larger, no significant improvement on the θ_{13} limit over current bounds was found in [5]. This stems mainly from the fact that in these experiments one is looking to constrain θ_{13} using the very small deviation of P_{ee} from unity. In other words, one is trying to differentiate between two scenarios, both of which predict a large number of events, differing from each other by a small number due to the small value of θ_{13}. Also, since $sgn(\Delta m^2_{31})$, which determines the neutrino mass ordering, is ascertained using earth matter effects, there is no hierarchy sensitivity in these survival channel experiments due to the smallness of this effect at the short baselines involved.

In this letter, we emphasise on the existence of large matter effects in the survival channel P_{ee} for an experiment with a very long baseline. Recalling that $P_{ee} = 1 - P_{e\mu} - P_{e\tau}$ and since for a given $sgn(\Delta m^2_{31})$ *both* $P_{e\mu}$ and $P_{e\tau}$ will either increase or decrease in matter, the change in P_{ee} is almost twice that in either of these channels. Using the multi-GeV ν_e flux from a β-beam source, we show that this large matter effect allows for significant, even maximal, deviation of P_{ee} from unity. This, we demonstrate, can be a convenient tool to explore θ_{13}. This is in contrast to the reactor option or the β-beam experimental set-up in [5], where increasing the neutrino flux and reducing the systematic uncertainties are the only ways of getting any improvement on the current θ_{13} limit. We further show, for the first time, that very good sensitivity to the neutrino mass ordering can also be achieved in the P_{ee} survival channel owing to the large matter effects. We discuss plausible experimental set-ups with the survival channel and show how the large matter effect propels this channel, transforming it into a very useful tool to probe $sgn(\Delta m^2_{31})$ and θ_{13} even with relatively large room for systematic uncertainties.

To keep the discussion simple, we start with one mass scale dominance (OMSD) and the constant density approximation. OMSD implies setting $\Delta m^2_{21} = 0$. Under these conditions, P_{ee} in matter, for neutrinos of energy E and traveling through a distance L, can be expressed as,

$$P_{ee} = 1 - \sin^2 2\theta^m_{13} \sin^2 \left[1.27(\Delta m^2_{31})^m L/E\right], \quad (1)$$

where the mass squared difference and mixing angle in matter are respectively

$$\begin{aligned} (\Delta m^2_{31})^m &= \sqrt{(\Delta m^2_{31}\cos 2\theta_{13} - A)^2 + (\Delta m^2_{31}\sin 2\theta_{13})^2} \\ \sin 2\theta^m_{13} &= \sin 2\theta_{13}\ \Delta m^2_{31}/(\Delta m^2_{31})^m \end{aligned} \quad (2)$$

and $A = 2\sqrt{2}G_F n_e E$ originates from the matter potential. Here, n_e is the ambient electron density. From Eq.

(1), we note that the largest deviation of P_{ee} from unity is obtained when the conditions (i) $\sin^2 2\theta_{13}^m = 1$ and (ii) $\sin^2\left[1.27(\Delta m_{31}^2)^m L/E\right] = 1$ are satisfied simultaneously. The first condition is achieved at resonance which is obtained for $A = \Delta m_{31}^2 \cos 2\theta_{13}$. This defines the resonance energy as,

$$E_{res} = \Delta m_{31}^2 \cos 2\theta_{13}/2\sqrt{2} G_F n_e. \tag{3}$$

The second condition gives the energy where the $(\Delta m_{31}^2)^m$ driven oscillatory term is maximal,

$$E_{max}^m = \frac{1.27(\Delta m_{31}^2)^m L}{(2p+1)\pi/2},\ p = 0, 1, 2.. \tag{4}$$

Maximum matter effect is obtained when $E_{res} = E_{max}^m$ [6]. This determines the distance for maximal matter effects as

$$(\rho L)^{max} = \frac{(2p+1)\pi\ 5.18\times 10^3}{\tan 2\theta_{13}}\ \text{km gm/cc}. \tag{5}$$

ρ denotes the electron density in gm/cc. This is the distance where $P_{ee} \simeq 0$ and the deviation of P_{ee} from unity is largest. We note that although both E_{res} and E_{max}^m depend on the value of Δm_{31}^2, the distance at which we get the maximum matter effect is independent of Δm_{31}^2. However, it is controlled by the value of θ_{13} very sensitively. For average earth matter densities obtained using the PREM density profile, one can find the typical distances at which the above conditions are satisfied for various values of $\sin^2 2\theta_{13}$ [7]. For instance for $p = 0$ and $\sin^2 2\theta_{13} = 0.2$ and 0.1, these distances are 7600 km and 10200 km respectively. For higher values of p the distance exceeds the earth's diameter for θ_{13} in the current allowed range. Using the value of $(\rho L)^{max}$ corresponding to the PREM density profile, from Eq. (5) one can estimate that the condition of maximal matter effects inside the earth's mantle is satisfied only for $\sin^2 2\theta_{13} \gtrsim 0.09$.

Under OMSD, the conversion probabilities in matter are

$$P_{ex} = Y_{23}\, \sin^2 2\theta_{13}^m \sin^2\left[1.27(\Delta m_{31}^2)^m L/E\right], \tag{6}$$

where $Y_{23} = \sin^2\theta_{23}$ for $x = \mu$ and $Y_{23} = \cos^2\theta_{23}$ for $x = \tau$. The maximum matter effect condition in the conversion channels is also given by Eq. (5). However, for the conversion probabilities there are suppression factors, $\sin^2\theta_{23}$ for $P_{e\mu}$ and $\cos^2\theta_{23}$ for $P_{e\tau}$, not present in P_{ee}. Moreover, since P_{ee} does not contain θ_{23}, the octant ambiguity as well as the effects of parameter correlations due to uncertainty in the allowed range of θ_{23} will be absent here. In addition, as mentioned earlier, the P_{ee} channel does not contain the CP phase, δ_{CP}. Both of these remain true in the presence of non-zero Δm_{21}^2 [8]. In our numerical work, we solve the full three flavour neutrino propagation equation assuming the PREM [9] density profile for the earth and keep Δm_{21}^2 and $\sin^2\theta_{12}$ fixed at their present best-fit values of 8.0×10^{-5} eV^2 and 0.31 respectively [10]. In what follows, we will also assume the true value of $\Delta m_{31}^2 = 2.5\times 10^{-3}$ eV^2.

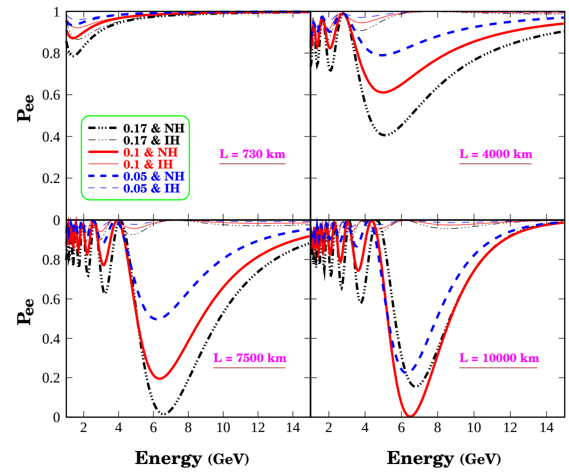

FIG. 1: P_{ee} in matter plotted versus neutrino energy. Thick (thin) lines are for normal (inverted) and hierarchy corresponding to $\pm\,|\Delta m_{31}^2|$ for four different baselines.

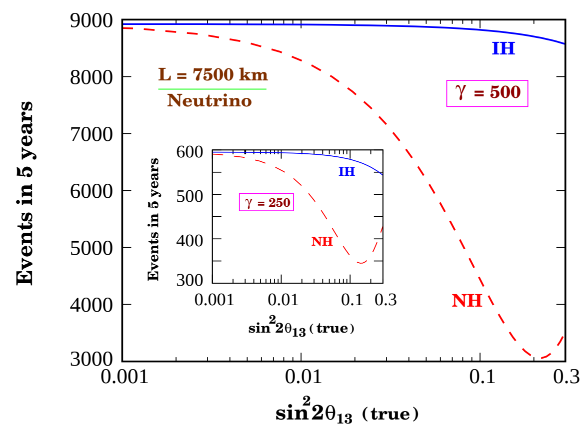

FIG. 2: Expected number of events in 5 years as a function of $\sin^2 2\theta_{13}$ for the normal (dashed line) and inverted (solid line) hierarchy for $L = 7500$ km and $\gamma = 500$. The inset shows the same but for $\gamma = 250$.

In Fig. 1 we plot P_{ee} as a function of energy, at four different L and for three values of $\sin^2 2\theta_{13}$. The plots confirm that maximal matter effects come at $L \simeq 10000$ km and $L \simeq 7500$ km for $\sin^2 2\theta_{13} = 0.1$ and 0.17 respectively for the normal hierarchy (NH). For the inverted hierarchy (IH) there is no significant matter effect for ν_e. This large difference in the probabilities for NH and IH can be exploited for the determination of the ordering of the mass levels. Further, since the matter effect is a sensitive function of θ_{13} it may also be possible to obtain information on this angle. We can also see that for a given value of $\sin^2 2\theta_{13}$ ($\gtrsim 0.09$) and E, matter effects increase (almost linearly) with L, until the L for which maximal matter effects can be achieved is reached, beyond which matter effects fall. For values of $\sin^2 2\theta_{13} < 0.09$ the condition for maximum matter effect is not met inside the earth's mantle and hence the matter effect and sensitivity to both hierarchy as well as θ_{13} increase with L.

In what follows, we will show how, in a plausible experiment, one can use this near-resonant matter effect in the survival channel P_{ee} to constrain θ_{13} and $sgn(\Delta m^2_{31})$. In addition to having a ν_e beam for probing this channel, Fig. 1 shows that the requirements for such a program include baselines of at least a few thousand km and average energies around 6 GeV. The detector should be able to observe e^- unambiguously at these energies.

Pure $\nu_e/\bar{\nu}_e$ beams can be produced from completely ionized radioactive ions accelerated to high energy decaying through the beta process in a storage ring, popularly known as β-beams [11, 12, 13]. The ions considered in the literature as possible sources for beta beams are ^6He and ^8B [14] for ν_e and ^{18}Ne and ^8Li for $\bar{\nu}_e$. The end point energies of ^6He and ^{18}Ne are $\sim$ 3.5 MeV while for ^8B and ^8Li this can be larger $\sim$ 13-14 MeV. For the Lorentz boost factor $\gamma = 250(500)$ the ^6He and ^{18}Ne sources have peak energy around $\sim 1(2)$ GeV whereas for the ^8B and ^8Li sources the peak occurs at a higher value around $\sim 4(7)$ GeV. Since the latter is in the ball-park of the energy necessary for near-resonant matter effects as discussed above, we will work with ^8B (^8Li) as the source ion for the ν_e ($\bar{\nu}_e$) β-beam and $\gamma = 250$ and 500. A β-beam facility could come up in the future at CERN, where $\gamma \simeq 250$ should be possible with the existing SPS, while $\gamma \leq 500$ could be produced with upgrades of the existing accelerators. The Tevatron at FNAL is also being projected as a plausible accelerator for the β-beam.

Water Čerenkov detectors have excellent capability of separating electron from muon events. Since this technology is very well known, megaton water detectors are considered to be ideal for observing β-beams. Such detectors do not have any charge identification capacity. But in a β-beam, the β^- and β^+ emitters can be stacked in different bunches and the timing information at the detector can help to identify the e^- and e^+ events [15].

It is well known that there are no beam induced backgrounds for β-beams. In this experimental set-up, the process $\nu_e \to \nu_\tau \to \tau^- \to e^-$ could mimic the signal. We have checked that the background to signal ratio for these events in the relevant energy range is $\sim 10^{-2}$ and can be neglected for the disappearance mode. e^- events from K and π^- decays are also negligible. The atmospheric background can be estimated in the beam off mode and reduced through directional, timing, and energy cuts.

Proposals for megaton water detectors include UNO [16] in USA, HyperKamiokande [17] in Japan and MEMPHYS [18] in Europe. If the β-beam is produced at CERN, then baselines in the range 7000-8600 km would be possible at any of the proposed locations for the UNO detector. Likewise, if the β-beam source be at FNAL, then the far detector MEMPHYS would allow for $L = 7313$ km. HyperKamiokande could also be considered as the far detector and in that case $L = 10184$ km if the source be at FNAL and $L = 9647$ km if it be at CERN.

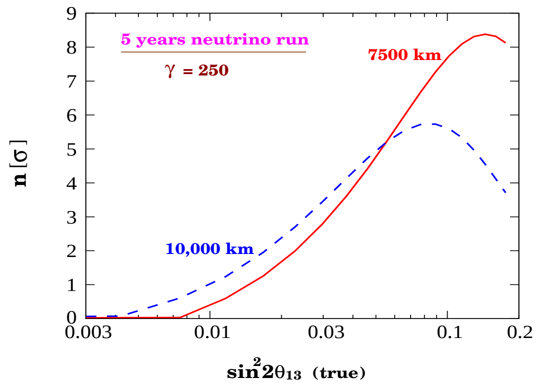

FIG. 3: Sensitivity to hierarchy for $L = 7500$ (solid line) and 10000 km (dashed line) and $\gamma = 250$, as a function of true value of $\sin^2 2\theta_{13}$.

For our numerical analysis we use the standard χ^2 technique with total χ^2 defined as, $\chi^2_{total} = \chi^2_{pull} + \chi^2_{prior}$, where $\chi^2_{prior} = \left[(|\Delta m^2_{31}| - |\Delta m^2_{31}(\text{true})|)/\sigma(\Delta m^2_{31})\right]^2$. In χ^2_{pull} we consider 2% β-beam flux normalisation error and 2% error for detector systematics. For the full definition of χ^2_{pull} and details of our statistical analysis, we refer to [19]. The prospective "data" is generated at the "true" values of oscillation parameters, assuming 440 kton of fiducial volume for the detector with 90% detector efficiency, threshold of 4 GeV and energy smearing of width 15%. For the ν_e β-beam we have assumed 2.9×10^{18} useful ^8B decays per year and show results for 5 years of running of this beam. The number of events expected as a function of $\sin^2 2\theta_{13}$ at $L = 7500$ with a $\gamma = 500$ ν_e β-beam is shown in Fig. 2 for the normal and inverted hierarchy. The inset in Fig. 2 shows the corresponding number of events in 5 years expected from a lower $\gamma = 250$. We have used the neutrino-nucleon interaction cross-sections from [20].

In Fig. 3 we show the sensitivity ($n\sigma, n = \sqrt{\chi^2}$) of the survival channel to the neutrino mass ordering for $L = 7500$ and 10000 km and $\gamma = 250$. If the true value of $\sin^2 2\theta_{13} = 0.05$, then one can rule out the inverted hierarchy at the 4.8σ (5.0σ) level with $L = 7500$ (10000) km. For $L = 7500$ (10000) km, the wrong inverted hierarchy can be disfavored at the 90% C.L. if the true value of $\sin^2 2\theta_{13} > 0.03$ (0.025). The sensitivity of the experiment improves significantly if we use $\gamma = 500$ instead of 250, since (i) the flux at the detector increases, and (ii) the flux peaks at E closer to 6 GeV, where we expect largest matter effects [19]. For $\gamma = 500$, the inverted hierarchy can be disfavored at $2.6\sigma(3.8\sigma)$ for a lower value of $\sin^2 2\theta_{13} = 0.015$. Minimum values of $\sin^2 2\theta_{13}$ at which the inverted hierarchy can be ruled out at 90% and 3σ

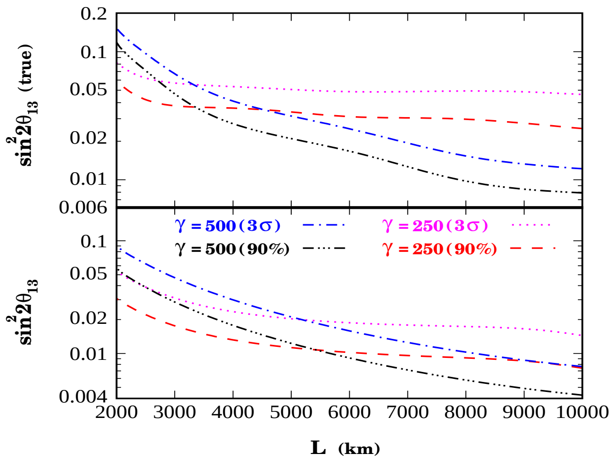

FIG. 4: The upper panel shows smallest value of true $\sin^2 2\theta_{13}$ at which normal and inverted hierarchy can be distinguished while the lower panel gives the sensitivity to $\sin^2 2\theta_{13}$ at various baselines at 90% and 3σ C.L., for two values of γ.

C.L. for different values of L are shown in the upper panel of Fig. 4 for both $\gamma = 250$ as well as $\gamma = 500$. From the figure one can see that for instance for $\gamma = 500$ and $L = 7500$ (10000) km the wrong inverted hierarchy can be disfavored at the 90% C.L. if the true value of $\sin^2 2\theta_{13} > 1.0 \times 10^{-2}$ (8.0×10^{-3}).

If the true value of $\sin^2 2\theta_{13}$ turns out to be smaller than the sensitivity reach shown in the upper panel of Fig. 4 for a given L, then it would not be possible to determine the hierarchy at the given C.L. However, we would still be able to put better constraints on $\sin^2 2\theta_{13}$ itself. The lower panel of Fig. 4 demonstrates as a function of L the sensitivity to θ_{13}, i.e., the minimum value of $\sin^2 2\theta_{13}$ which can be statistically distinguished from $\sin^2 2\theta_{13} = 0$ at 90% and 3σ C.L. Both Figs. 3 and 4 show that the sensitivity improves with L, even though the flux falls as $1/L^2$. This results from matter effects increasing with L, as pointed out before. For $L = 7500$ (10000) km, we can constrain $\sin^2 2\theta_{13} < 6.3 \times 10^{-3}$ (4.3×10^{-3}) at the 90% C.L. for $\gamma = 500$.

So far, we have used only the total rates in our analysis. If the energy can be reconstructed accurately, then the result can be improved further. For instance, for L=7500 km, if one could preferentially select the energy in the range 5.0-7.5 GeV, then the normal and inverted hierarchy would be differentiated for $\sin^2 2\theta_{13} = 7.47 \times 10^{-3}$ at 90% C.L. for $\gamma = 500$.

We have presented our results using a ν_e beam and assuming NH to be the true hieararchy. Similar results can also be obtained with a $\bar{\nu}_e$ beam for IH. It is also possible to run both beams simultaneously.

In conclusion, we propound the possibility of using large matter effects in the survival channel P_{ee} at long baselines for determination of the neutrino mass ordering ($sgn(\Delta m^2_{31})$) and the yet unknown leptonic mixing angle θ_{13}. Matter effects in the transition probabilites $P_{e\mu}$ and $P_{e\tau}$ act in consonance to give an almost two-fold effect in the survival channel. In addition, the problem of spurious solutions due to the leptonic CP phase and the atmospheric mixing angle θ_{23} does not crop up. The development of β-beams as sources of pure $\nu_e/\bar{\nu}_e$ beams enables one to exclusively study the P_{ee} survival probability. The sensitivity to $sgn(\Delta m^2_{31})$ and θ_{13} that is obtained through this channel is comparable to that of most of the proposed accelerator based future experiments involving the "golden channel".

We thank F. Terranova for a useful communication.

[1] A. Cervera *et al.*, Nucl. Phys. B **579**, 17 (2000) [Erratum-*ibid.* B **593**, 731 (2001)].
[2] J. Burguet-Castell *et al.* Nucl. Phys. B **608**, 301 (2001); H. Minakata and H. Nunokawa, JHEP **0110**, 001 (2001); G. L. Fogli and E. Lisi, Phys. Rev. D **54**, 3667 (1996); V. Barger, D. Marfatia and K. Whisnant, Phys. Rev. D **65**, 073023 (2002).
[3] D. Autiero *et al.*, Eur. Phys. J. C **33**, 243 (2004); A. Donini, D. Meloni and P. Migliozzi, Nucl. Phys. B **646**, 321 (2002).
[4] K. Anderson *et al.*, arXiv:hep-ex/0402041.
[5] A. Donini, E. Fernandez-Martinez and S. Rigolin, Phys. Lett. B **621**, 276 (2005).
[6] R. Gandhi *et al.*, Phys. Rev. Lett. **94**, 051801 (2005).
[7] R. Gandhi *et al.*, Phys. Rev. D **73**, 053001 (2006).
[8] E. K. Akhmedov *et al.*, JHEP **0404**, 078 (2004).
[9] A. M. Dziewonski and D. L. Anderson, Phys.Earth Planet.Interiors **25**, 297 (1981).
[10] M. Maltoni *et al.*, New J. Phys. **6**, 122 (2004); S. Choubey, arXiv:hep-ph/0509217; S. Goswami, Int. J. Mod. Phys. A **21**, 1901 (2006).
[11] P. Zucchelli, Phys. Lett. B **532**, 166 (2002); For a recent review see C. Volpe, J. Phys. G **34**, R1 (2007).
[12] Physics potential of β-beams for baselines $\leq$ 3000 km is studied e.g. in P. Huber *et al.*, Phys. Rev. D **73**, 053002 (2006); J. Burguet-Castell *et al.*, Nucl. Phys. B **725**, 306 (2005); J. Burguet-Castell *et al.*, *ibid.* **695**, 217 (2004).
[13] Physics potential of β-beams for very long baselines is studied in S. K. Agarwalla, A. Raychaudhuri and A. Samanta, Phys. Lett. B **629**, 33 (2005); R. Adhikari, S. K. Agarwalla and A. Raychaudhuri, *ibid.* **642**, 111 (2006); S. K. Agarwalla, S. Choubey and A. Raychaudhuri, arXiv:hep-ph/0610333.
[14] C. Rubbia *et al.*, Nucl. Instrum. Meth. A **568**, 475 (2006); C. Rubbia, arXiv:hep-ph/0609235.
[15] A. Donini *et al.* arXiv:hep-ph/0604229.
[16] C. K. Jung, AIP Conf. Proc. **533**, 29 (2000).
[17] Y. Itow *et al.*, arXiv:hep-ex/0106019.
[18] L. Mosca, Nucl. Phys. Proc. Suppl. **138**, 203 (2005).
[19] The last reference in [13].
[20] P. Huber, M. Lindner and W. Winter, Comput. Phys. Commun. **167**, 195 (2005).